A NOVEL METHOD OF DENOISING NATURAL IMAGES USING CURVELETS COMBINING WITH CYCLE SPINNING

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Abstract: The purpose of this paper is to develop a method for denoising natural images corrupted with random noise. The noise degrades quality of the images and makes interpretations, analysis and segmentation of images tougher. In the paper the use of the cycle spinning with curvelet transform for noise reduction is considered. The curvelet transform is a new image representation approach that codes image edges more efficiently than the wavelet transform. Edges are very important in image perception and with fewer coefficients to represent edges; a better denoising scheme can be achieved. By use of this technique one can able to get better PSNR values and lesser MSE values.
Keywords: Thresholding, Cycle Spinning, PSNR (Peak Signal Noise ratio), MSE (Mean square error), UQI (Universal quality Index), PCC (Pearson Correlation Coefficient).

I. INTRODUCTION

In today's life, digital images are used in various applications. During image acquisition or transmission process digital images are usually corrupted by noise. A variety of factors, such as environmental conditions during image acquiring, and the quality of the sensing elements themselves affect the performance of imaging sensors. The noise in images is introduced by a charge-coupled device (CCD) camera used for acquiring images, light levels and sensor temperature. Images are corrupted during transmission process also due to interference in the transmission channel.

Denoising is an important and pre-processing stage in the Image Enhancement process. Denoising is necessary and first step to be taken before the image data is analyzed for further use. Because noise destroys the important details and features present in the image. It is necessary to apply efficient and effective denoising technique to compensate for such data loss.

A time invariant version of the fast discrete curvelet transform is proposed by implementing cycle spinning on two of the three sub-bands of the curvelet transform. The biggest problem with image denoising is the edges of images. By performing cycle spinning the largest error of a denoised image is reduced resulting in lower energy of the error which gives better denoising result [13].

1.1 Noise Model

Let the original image 'f' where $f = \{f_{ij}, i, j = 1,2, \dots M\}$ denote the $M \times M$ matrix of the original image (noise free) and 'M' is some integer power of 2. The original image has been corrupted by additive noise and resulted image is represented as:

$$g_{ij} = f_{ij} + \sigma n_{ij}, \quad \dots\text{(i)}$$

Where, 'n_{ij}' is a standard random noise, 'σ' is the noise level or standard deviation and 'g_{ij}' is the noisy image. The goal is to remove the noise, or "denoise"$\{g_{ij}\}$, and to estimate the image 'f' from noisy image 'g_{ij}' such that Mean Squared Error (MSE) is minimum and Peak Signal Noise Ratio (PSNR) is maximum.

The paper is organized as follows. First, in Section 2, the Wavelet Transform, its pros and cons are introduced. The curvelet transform is discussed in Section 3 and the different denoising methods with proposed method (Cycle spinning combined with Curvelet Transform via wrapping) are described in Section 4. Experimental results are given in Section 5 and finally, conclusions are drawn in Section 6.

II. WAVELET TRANSFORM

Wavelet transform consists of a set of functions that can be used to examine signals (image) in both time and frequency domains simultaneously. This analysis is accomplished by the use of a scalable window to cover the time-frequency plane, providing a suitable means for the analyzing of non-stationary signal that is often found in most application. Wavelet analysis uses a wavelet prototype function known as the mother wavelet. This mother wavelet in turns generates a set of functions known as child wavelets during recursive scaling and translation. The variable's' reflects the scale or width of a basis function and the variable 'τ' is the translation that specify its translated position on the time axis:

$$\Psi(\tau, s) = \frac{1}{\sqrt{s}}\,\Psi\left(\frac{t-\tau}{s}\right) \quad \dots\text{ (ii)}$$

Where, '$\Psi\left(\frac{t-\tau}{s}\right)$' the mother wavelet and the factor '$\frac{1}{\sqrt{s}}$' is a normalized factor used to ensure energy across different scale remains the same. Wavelets are good at catching zero-dimensional or point singularities, but two dimensional piecewise smooth signals resembling images have one dimensional singularities [8].

2.1 Drawback of Wavelet Transform (WT)

Wavelet Transform is not translation invariant. The decomposition process results in a multiresolution pyramid structure, does not maintain translation invariance property which means that a small shift in an image can cause a major variation in the distribution of energy of the wavelet coefficients at different levels and mild ringing artifacts around the edges. Discontinuities across a simple curve affect all the wavelets coefficients on the curve. Hence the WT doesn't handle curves discontinuities well.

III. CURVELETS TRANSFORM (CT)

Curvelet Transform is a new multi-scale representation which is most suitable for objects with curves. CT developed by Candès and Donoho (1999). A discontinuity point affects all the Fourier coefficients in the domain. Hence the FT doesn't handle point's discontinuities well. Using wavelets, it affects only a limited number of coefficients. Hence the WT is able to handle point discontinuities well. Discontinuities across a simple curve affect all the wavelets coefficients on the curve. Therefore the WT doesn't handle curves discontinuities well. Curvelets are designed to handle curves using only a small number of coefficients. That's why the CT handles curve discontinuities well. Candes and Donoho [9], [11] proposed another multiscale transform called Curvelet transform which is designed to handle curve discontinuities well. Here, the idea is to partitioning the curves into collection of ridge fragments and then handle each fragment using the ridgelet transform.

The basic process of Curvelet Transform consist the following four stages:

- Sub-band decomposition: First define a bank of low pass filter P0 and band pass filters $\Delta_s, s \geq 0$ such that the image f is filtered into sub-bands with à trous algorithm

$$f \longmapsto (P_0 f, \Delta_1, \Delta_2, \ldots) \quad \ldots \text{(iii)}$$

- Smooth Partitioning: Each sub-band is smoothly windowed into dyadic squares

$$h_Q = w_Q . \Delta_s f \, , \forall Q \in Q_s \quad \ldots \text{(iv)}$$

where w_Q is collection of smooth windowing function localized near Q_S

$$Q_{(s,k_1,k_2)} = \left[\frac{k_1}{2^s}, \frac{k_1+1}{2^s}\right] \times \left[\frac{k_2}{2^s}, \frac{k_2+1}{2^s}\right] \quad \ldots \text{(v)}$$

- Renormalization: Each dyadic square is centering to the unit scale $g_Q = T_Q^{-1} . h_Q$,where

$$(T_Q f)(x_1, x_2) = 2^s f(2^s x_1 - k_1, 2^s x_2 - k_2) \quad \dots \text{(vi)}$$

is the renormalization operator.

- Redgelets Analysis: Each normalized square in the ridgelet system

$$\alpha_{(Q,\lambda)} = \langle g_Q, \rho_\lambda \rangle \quad \dots \text{(vii)}$$

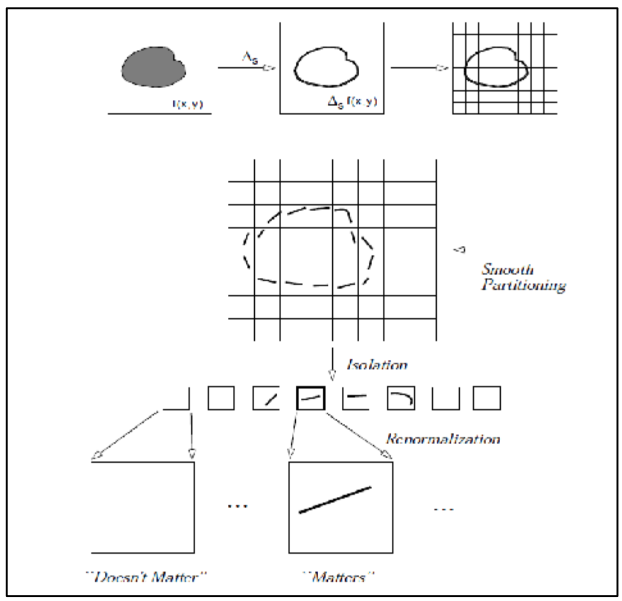


Fig:1 Example of different stages of CT

The curvelets are based on multiscale ridgelets collective with a spatial bandpass filtering operation. These bandpass filters are set so that the curvelet length and width at fine scale obey the scaling rule width   length 2. A 2-D wavelet transform is used to isolate the image at different scales and spatial partitioning is used to break each scale into blocks. Large size blocks are used to partition the large scale wavelet transform components and small size blocks are used to partition the small scale components. Finally, the ridgelet transform is applied to each block. In this way, the image edges at a certain scale can be represented efficiently by the ridgelet transform because the image edges are almost like straight lines at that scale. The Curvelet transform can sparsely characterize the high-dimensional signals which have lines, curves or hyper plane singularities. [1] The Curvelet transform is a higher dimensional generalization of the wavelet transform deliberate to represent images at different scales and different angles. Curvelet transform is an exceptional member of the multi-scale geometric transforms. It is a transform with multi-scale pyramid with lots of directions at each length scale. Curvelets will be superior over wavelets in following cases:

- Optimally sparse representation of objects with edges
- Optimal image reconstruction in severely ill-posed
- Optimal sparse representation of wave propagators

The discrete curvelet transform (DCT) takes as input a Cartesian grid of the form $f(n_1, n_2)$, $0 \le n_1, n_2 < n$, and output of collection of coefficients $c^D(j,l,k)$ defined by

$$c^D(j,l,k) = \sum_{n_1,n_2} f(n_1,n_2)\overline{\varphi^D_{j,l,k}(n_1,n_2)} \qquad \ldots \text{(viii)}$$

where $\varphi^D_{j,l,k}(n_1,n_2)$ are digital curvelet waveforms which maintain the listed properties of the continuous curvelet.

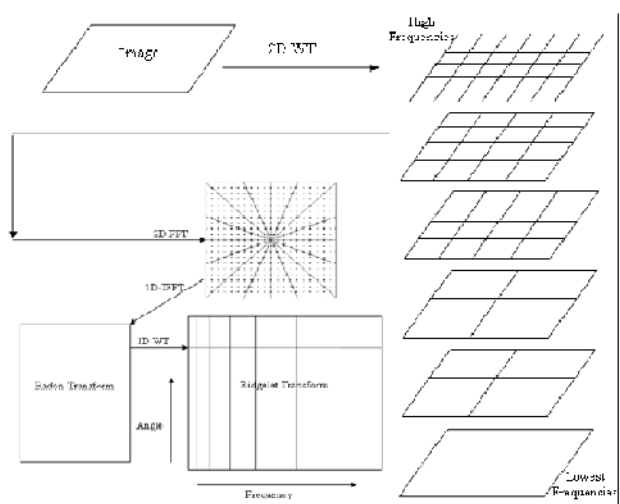


Fig:2 Block diagram of Curvelet Transform

CT can be implemented in 2 ways (FDCT via USFFT and FDCT via wrapping), which differ by spatial grid used to translate curvelets at each scale and angle. In this paper FDCT via wrapping is implemented as it is simpler, faster and less redundant. This transform is constructed using parabolic scaling, anisotropic law, tight framing and wrapping.

For the 2D image, the architecture of the DCT via wrapping is as follows:

- Apply the 2D FFT and obtain Fourier samples

$$\hat{f}[n_1, n_2], \; -n/2 \le n_1, n_2 < n/2. \qquad \text{… (ix)}$$

- For each scale j and angle l, form the product

$$\tilde{U}_{j,l}[n_1, n_2]\hat{f}[n_1, n_2]. \qquad \text{… (x)}$$

- Wrap this product around the origin and obtain

$$\tilde{f}_{j,l}[n_1, n_2] = W(\tilde{U}_{j,l}\hat{f})[n_1, n_2]. \qquad \text{… (xi)}$$

- Apply the inverse 2D FFT to each $\tilde{f}_{j,l}[n_1, n_2]$, hence collecting the discrete coefficients

$$c^D(j,l,k). \qquad \text{… (xii)}$$

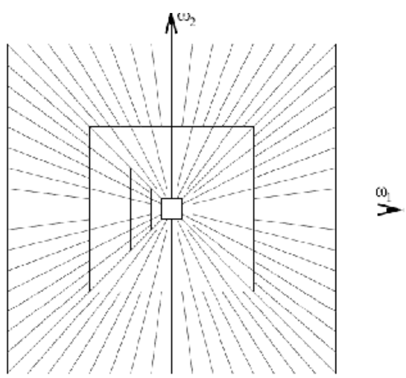


Fig:3 Curvelet tiling, also called second dyadic decomposition of the frequency plane. [6]

IV. DENOISING METHODS:

In this paper we have discussed mainly three denoising methods. First is VisuShrink and second is BayesShrink, both are wavelets based denoising method. Third method is proposed method which is based on Curvelet Transform via wrapping combining with Cycle spinning.

4.1 VisuShrink

VisuShrink was introduced by Donoho . The threshold value t, is derived from the standard deviation of the noise. It uses hard thresholding rule. It is also known as universal threshold and t is defined as $t = \sigma\sqrt{2 \log n}$. σ^2 is the noise variance present in the signal and n represents the signal size or number of samples. An estimate of the noise level σ was defined based on the median absolute deviation given by

$$\hat{\sigma} = \frac{median(\{|g_{j-1k}|: k = 0,1,\ldots,2^{j-1}-1\})}{0.6745} \quad \ldots \text{(xiii)}$$

where g_{j-1k} is refers to the detail coefficients in the wavelet transform. The main drawback of VisuShrink is it does not deal with minimizing the mean squared error. However, VisuShrink gives the images that are overly smoothed. This is because VisuShrink removes too many coefficients. Another drawback is that it cannot remove speckle noise, which is multiplicative noise. It can only handle an additive noise. VisuShrink follows the global thresholding method; here global threshold means a single value of threshold applied globally to all the wavelet coefficients.

4.2 BayesShrink (BS)

BayesShrink was proposed by Chang, Yu and Vetterli. The goal of this method is to minimize the Bayesian risk, and hence its name, BayesShrink. It uses soft thresholding Wavelet shrinkage is a method of removing noise from images in wavelet shrinkage, an image is subjected to the wavelet transform, the wavelet coefficients are found, the components with coefficients below a threshold are replaced with zeros, and the image is then reconstructed[10]. The Bayes threshold t_b, is defined as

$$t_b = \frac{\sigma^2}{\sigma_s} \quad \ldots \text{(xiv)}$$

where σ^2 is the noise variance and σ_s^2 is the signal variance without noise. The BS method is effective for images including Gaussian noise. The observation model is expressed as follows: V =X + Y. Here Y is the wavelet transform of the degraded image, X is the wavelet transform of the original image, and V denotes the wavelet transform of the noise components following the Gaussian distribution $N(0,\sigma_v^2)$. Here, since X and V are mutually independent, The variances σ_y^2, σ_x^2 and σ_v^2 of y, x and v is given by $\sigma_y^2 = \sigma_x^2 + \sigma_v^2$.

4.3 Curvelets denoising using cycle spinning (Proposed Method)

To denoise the image and to enhance the details present in the image and to improve the results of curvelet transform, the concept of cycle spinning combined with curvelet transform (CTCS) is used for denoising.

In this case first cycle spinning is applied on noisy image within certain range to get a shifted image, which has some phase shift compared with the noisy image. Then curvelet transform via wrapping is used to obtain curvelet coefficients g_Y^{CT}. Then Block thresholding at finest scale k=2.3 and at all scale except finest scale k = 2.2 is applied to coefficients g_Y^{CT}. After applying inverse curvelet transform, the denoised image is processed by inverse cycle spinning to obtain the denoised image whose phase is the same as the noisy image. This process is repeated by changing shift in the range of circulated shifts. In conclusion, all the obtained results are averaged to get the final denoised image.

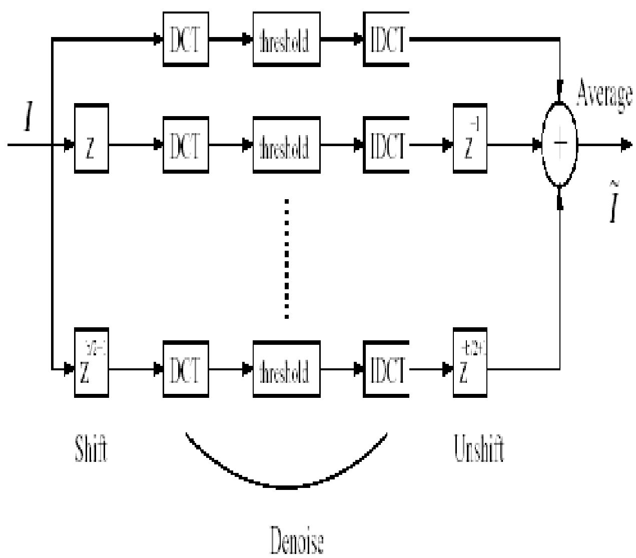


Fig:4 Framework of the cycle spinng denoising algorithm. Here DCT stands for discrete curvelet transform , I is Noisy Image , $\tilde{I}$ is denoised Image and b stands for sidelength.

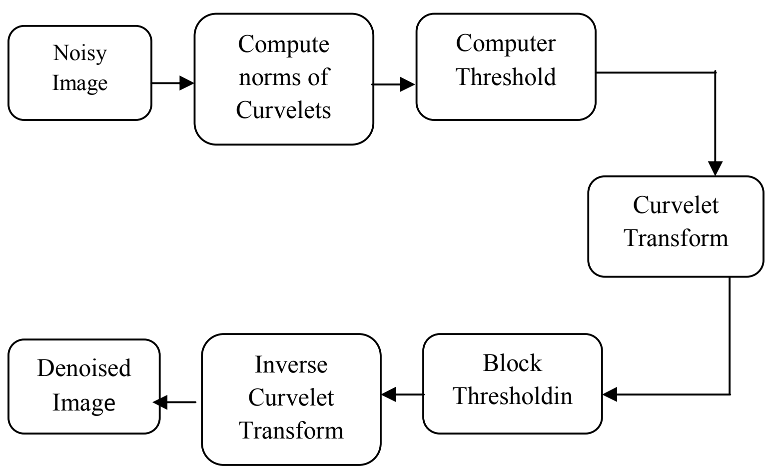


Fig:5 Framework of the FDCT denoising algorithm.

V. EXPERIMENTAL RESULTS

(a) (b) (c)

(d) (e)

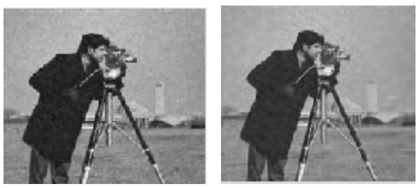
Fig:6 (a) Lena's Original Image.(b) Nosiy Image with σ= 20. (c) Denoised image using VisuShrink. (d)Denoised image using BayesShrink. (e)Denoised using Proposed method.

Metric -- ----------- Denoising Method	**PSNR**	**MSE**	**UQI**	**MSSIM**	**PCC**
VisuShrink	24.17	248.7	0.53	0.72	6.27
Bayes Shrink	28.48	92.13	0.64	0.79	6.42
Proposed Method	29.77	84.95	0.68	0.83	6.45

Table : 1 Shows the values of performance metrics like PSNR (Peak Signal Noise ratio), MSE (Mean square error), UQI (Universal quality Index), MSSIM (mean Structural

Similarity),PCC (Pearson Correlation Coefficient) after applying different denoising methods on Lena’s image with sigma=20.

(a) (b) (c)



(d) (e)

Fig:7 (a) Cameraman’s Original Image. (b) Noisy Image with σ= 20. (c) Denoised image using VisuShrink. (d) Denoised image using BayesShrink. (e) Denoised using Proposed method.

Metric **Denoising Method**	**PSNR**	**MSE**	**UQI**	**MSSIM**	**PCC**
VisuShrink	24.25	244.3	0.54	0.69	6.36
Bayes Shrink	28.48	92.18	0.64	0. 757	6.46
Proposed Method	29.72	69.38	0.68	0. 795	6.49

Table : 2 Shows the values of performance metrics after applying different denoising methods on Cameraman’s image with sigma=20.

VI. CONCLUSION:

The curvelet transform combining with cycle spinning, the denoising method for natural images was proposed and compared with the VisuShrink and BayesShrink wavelets based denoising methods. All the three methods significantly reduce the random noise while preserving the resolution and the structure of the original images as is shown in Fig. 6 and Fig. 7. Experimental results given in Table 1 and Table 2 show that the Proposed method outperforms in all five estimations giving a good and clean image, which should improve details and recognition.

REFERENCES:

[1] H. S. Bhadauria, M. L. Dewal, R. S. Anand, "Comparative Analysis of Curvelet based techniques for denoising of Computed Tomography Images",2011.

[2] A.A. Patil,J. Singhai," Image denoising using curvelet transform: an approach for edge preservation", Journal of scientific and industrial research, Vol. 69,pp. 34-38, january2010.

[3] A.D. Ali, P. D. Swami and J. Singhai, "Modified Curvelet Thresholding Algorithm for Image Denoising", Journal of Computer Science 6 (1): pp. 18-23, 2010.

[4] K. Ding "Wavelets, Curvelets and Wave Atoms for Image Denoising," 3rd International Congress on Image and Signal Processing (CISP2010), 2010.

[5] N.T.Binh and A.Khare," Multilevel Threshold Based Image Denoising in Curvelet Domain", JOURNAL OF COMPUTER SCIENCE AND TECHNOLOGY, vol. 25(3), pp.632-640, May 2010.

[6] L. Demanet, L. Ying "Curvelets and Wave Atoms for Mirror-Extended Images," Proc. SPIE Wavelets XII conf, San Diego, August 2007.

[7] B. B. Saevarsson, J. R. Sveinsson and J. A. Benediktsson, "Time Invariant Curvelet Denoising", 6th Nordic Signal Processing Symposium – NORSIG, 2004.

[8] Minh N. Do and Martin Vetterli, The Finite Ridgelet Transform for Image Representation, IEEE Transactions on Image Processing, 12(1),pp. 16-28, January 2003.

[9] J.L. Starck, E. Candes, and D.L. Donoho, The Curvelet Transform for Image Denoising, IEEE Transactions on Image Processing, 11(6),pp. 670 -684, 2002.

[10] S. Grace Chang, "adaptive Wavelet Thresholding for Image denoising and compression," IEEE transactions on Image processing, Vol.9, No.9 september.2000.

[11] David L. Donoho & Mark R. Duncan, Digital Curvelet Transform: Strategy, Implementation and Experiments, Stanford University, November, 1999.

[12] D.L.Donoho, "Denoising and soft thresholding," IEEE. Transactions, Information, Theory, VOL.41, PP.613-627, 1995.

[13] R.R. Coifman and D.L. Donoho, “Translation-Invariant Denoising, ”Wavelets and Statistics, Springer Lecture notes in Statistics 103, pp. 125–150, New York: Springer-Verlag, 1995.